2008 International Seminar on Future Information Technology and Management Engineering

Empirical Analysis of Portfolio Optimization Based on DEA model

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Abstract

The asset evaluation process plays an important role in portfolio optimization because it is the prerequisite for investment decision making and directly influences on the asset allocation. This paper presents an evaluation analysis of stock investment for portfolio optimization based on Data Envelopment Analysis (DEA) model. Considering the relationship between portfolio return and risk, the efficient DEA models and relative evaluation criteria are established. The empirical analysis is conducted on twenty-seven stocks selected from five industrial sectors of China A-share stock market, and a comparative analysis is extended for the fundamental and weight-restricted DEA models.

1. Introduction

In capital market, the argument often heard is that the high expected return always implies a high risk. In a fully efficiency, the strategy in favor of the investment is that it lowers the risk without sacrificing the expected return, so it would be a better strategy to invest in a portfolio with risk diversification rather than a specific security. The asset diversification is regarded as one of most efficient approaches which can be realized in the practice of portfolio optimization[1].

In the process of portfolio optimization, the asset evaluation process plays an important role because it is the prerequisite for investment decision making and directly influences on the asset allocation. It has not drawn the domestic and foreign investor's attention to make deep insights into investment decision making for maximizing the return of portfolio. Compared to modern portfolio theory, researches on investment decision making are not systematic. The common approaches are Discounted Cash Flow (DCF), Value at Risk (VaR), and so on. The quantitative method of fuzzy mathematics has also been introduced in some literatures. These approaches do have their own merits and have been widely used to solve practical problems. However, they emphasize too much on the market value based on the historical financial instruments, ignoring the subjective factors that influence on the asset price. To overcome the problem that the investors allocate the assets subjectively without considering the efficiency of risk and return, the Data Envelopment Analysis (DEA) model can be adopted for this purpose.

DEA has been increasing its importance as a tool for evaluation and resource allocation in the fields of expert system and decision making. The applications can be found in the domestic and foreign publications. For examples, Hongyu Li[2], who did study on array performance analysis of Decision Making Unit (DMU) through combining the efficient fraction and relative efficient ratio; Antonella Basso[3] introduced DEA model into risk endurance assessment; Guangxi Cao[4] and Tienui Dong[5] used DEA model to analyze the performance of fund companies and security agencies; Yihua Chen[6] tried to utilize the model for improving the performance of input and output selection. All these model applications indicate that DEA is a flexible model and has a wide application. This paper presents an evaluation of portfolio optimization based on fundamental and weight-restricted DEA models, which are able to measure the portfolio performance comprehensively.

2. DEA modeling

Data Envelopment Analysis (DEA) is a multifactor productivity analysis model for measuring the relative efficiency of a homogenous set of decision making units (DMUs).

Supposed each DMU_j ($1 \le j \le n$) has m input indexes and s output indexes, and $x_j=(x_{1j}, x_{2j}, x_{3j}, \dots, x_{mj})^T > 0$, $y_j= (y_{1j}, y_{2j}, y_{3j}, \dots, y_{sj})^T > 0$. Let $\mathbf{X}=[x_1, x_2, x_3, \dots, x_n]$ be a multiple-index input matrix, and $\mathbf{Y}=[y_1, y_2, y_3, \dots, y_n]$ be a multiple-index output matrix. **V, U** represent input weight array and output weight array, respectively. The efficiency score in the presence of multiple input (I_j) and output (O_j) factors is defined as [7]:

$$E_{ii} = \frac{O_i}{I_i} = \frac{\sum_{i=1}^{s} u_i y_{1i}}{\sum_{i=1}^{m} v_i x_{1i}} = \frac{y_i^T u}{x_i^T v} \qquad (1)$$

Where $\mathbf{V} = [v_1, v_2, v_3, \dots, v_m]$ and $\mathbf{U} = [u_1, u_2, u_3, \dots, u_s]$ which are unknown.

2.1 Fundamental DEA model

To optimize the weights allocation, the problem can be express as:

978-0-7695-3480-0/08 $25.00 © 2008 IEEE
DOI 10.1109/FITME.2008.129

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$$\begin{cases} \max\ y_i^T u = E_{ii} \\ s.t. y_i^T u \le x_j^T\ v(j = 1,2,3,\cdots n) \\ x_j^T\ v = 1 \\ u \ge 0,\ v \ge 0 \end{cases} \tag{2}$$

Each DMU can find a solution u_i^* and v_i^* from (2) to maximize the efficient score $E_{ii}=y_i^T u^*$. If $E_{ii}=1$, DMU_i is considered as efficient; and if $E_{ii}<1$, DMU_i is considered as inefficient.

2.2 Weight-restricted DEA model

Taking all the input and output factors as same importance level in DEA model is not appropriate, so the weight-restricted constraints can be introduced for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. For example, if output 1 is at least twice as important as output 2 then this can be incorporated into the DEA model by using the linear constraint $v_1 \ge 2v_2$. The modified model can be expressed as:

$$\begin{cases} \max y_i^T u = E_{ii} \\ s.t. x_j^T v - y_i^T u \in K(j = 1,2,3...n) \\ x_j^T v = 1 \\ v \in V, u \in U \end{cases} \tag{3}$$

2.3 Value ranking and the criteria

DEA score indicates the relative investment value of the asset. In terms of the investing value, the portfolio should consist of high DEA score assets. However, a high return usually implies a high risk. To show the efficiency ranking of DMU, we set the ranking criteria as follows [8]:

- If E_{ii} in (2) or (3) is equal to 1, it is the first-class asset which has a high investment value.
- If $0.5 \le E_{ii} \le 1$, it is the second-class asset which is of some value for investing but need more consideration.
- If $E_{ii}<0.5$, the asset has no value for investing.

3. Empirical analysis

3.1 Input and output index

The inputs for each DMU are designed based on the following five aspects: the profit capability, the developing capability, the operation efficiency, the debt payment capability, and the market performance; the outputs contain the portfolio return and the risk. The indexes of input and output are listed in Table 1.

The volatility, which indicates the price fluctuation and measures the divergence extent of the stock price, is often calculated from the standard deviation.

The RSI (Relative Strengthen Index), reflecting the market's performance and oscillating in a range between 0 and 100, represents a comparison of the magnitude of a stock's recent gains to the magnitude of its recent losses. In mathematical terms [9],

$$RSI = 100 - \frac{100}{1 + RS} \tag{4}$$

where, RS is calculated as the ratio of two exponentially smoothed moving averages, AG/AL. AG is the average price gain over some period and AL is the average price drop over some the same period. The RSI period is supposed to be two weeks.

Table 1: The input and output of DMU

Inputs			Outputs		
Profit capability	X1	Earning/ share	Return	Y1	Return of portfolio
	X2	ROE	Risk	Y2	Volatility
Developing capability	X3	Growth rate of net income			
Operation efficiency	X4	Asset turnover ratio			
	X5	Ratio of asset/debt			
Debt-payment capability	X6	Liquidity ratio			
Market performance	X7	RSI			

3.2 Analysis of fundamental DEA model

The stock quote covers a time span from January 1, 2007 to December1, 2007. The data are downloaded from the web site: http://finance.sina.com/. Based on the DEA model and data sample, the evaluation results are listed in Table 2. In accordance with the evaluation criteria and ranking DEA score, fifteen of the twenty-seven stocks are in the first-class category; the other twelve stocks are second-class, which can be seen in Table 3. The result closely reflects the actual market performance.

In modern portfolio theory, asset selection is based on the principle of profit maximization under a given risk, or risk minimization under a given profit. To test weather the DEA model complies with this principle, we calculate the ratio of return and risk (r/σ) which can be seen in Table 4. It can be observed that the ratio of first-class stocks is higher than the second-class, which means investing the first-class stocks could make a higher return.

The conclusion can also be illustrated from Figure 1, which shows that the regression line of the first-class stocks is above the line of the second-class, the average return/risk ratio of the first-class is also greater than that of second-class. However, this observation does not exist for any case because the return/risk is a relative ratio and the using of standard deviation as risk is not so appropriate in some cases. The efficient stocks with a low the ratio of r/σ do not prove to be low value for investment.

Table 2: Result from DEA model

sector	stock name	score	sector	stock name	score
agriculture	XSGF	0.905	nonferrous metal	ZJHJ	1.000
	GTZL	0.890		ZSGF	1.000
	ZLTH	1.000		TYGY	1.000
	DFHY	0.755		ZKYH	1.000
	FLZY	0.881		TTGF	1.000
	average	0.886		average	1.000
real estates	WKA	0.883	electrical equipment	GDDL	1.000
	ZSDC	1.000		HNGJ	1.000
	JRJ	0.923		SZNY	1.000
	ZLDC	1.000		GDNZ	0.885
	QXJS	0.866		PGDQ	0.814
	BLDC	1.000		XDGF	1.000
	average	0.945		average	0.950
steel	HDGT	0.878			
	BGGF	1.000			
	WGGF	0.795			
	AGGF	1.000			
	LGGF	0.848			
	average	0.904			
average			0.938		

Table 3: Number of the first and second class stocks

Industry	first-class stocks	second-class stocks
Agriculture	1	4
real estates	3	3
Steel	2	3
nonferrous metals	5	0
electronic equipment	4	2
Total	15	12
Average score	1.000	0.860

Table 4: Return/risk ratio (r/σ)

The first-class stocks (E_{ii} =1)			The second-class stocks (E_{ii} <1)		
#	Stock name	r/σ	#	Stock name	r/σ
1	ZLTH	0.0944	1	XSGF	0.0952
2	ZSDC	0.0679	2	GTZL	0.1198
3	ZLDC	0.1051	3	DFHY	0.0765
4	BLDC	0.0261	4	FLZY	0.0667
5	BGGF	0.0790	5	WKA	0.0567
6	AGGF	0.1081	6	JRJ	0.0519
7	GDDL	0.1209	7	QXJS	0.0745
8	HNGJ	0.0949	8	HDGT	0.0654
9	SZNY	0.1207	9	WGGF	0.1158
10	XDGF	0.1139	10	LGGF	0.0810
11	ZJHJ	0.1769	11	GDNZ	0.0857
12	ZSGF	0.1477	12	PGDQ	0.0593
13	TYGY	0.1130			
14	ZKYH	0.0635			
15	TTGF	0.0732			
Average		0.1003	Average		0.0790

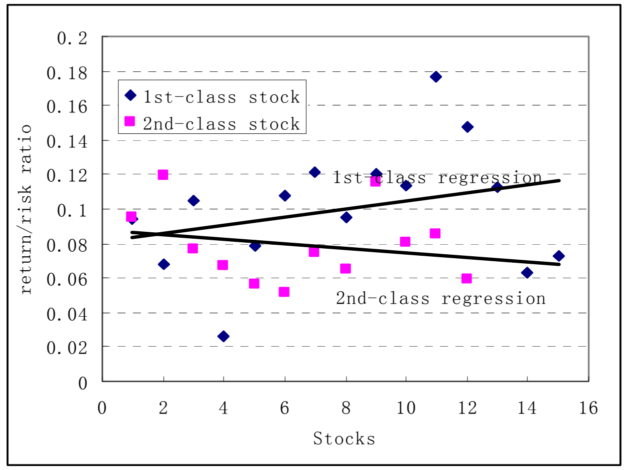

Figure 1: Comparison of the ratio(r/σ)

3.3 Analysis of weight-restricted DEA model

The weight-restricted DEA model has the advantage of taking the investor's risk preference into consideration and allocating weights for the input indicators in accordance with their importance. Combining the qualitative analysis with the results from fundamental DEA model, we assign the weights for the inputs as shown in Table 5. The outputs are weighed in return-oriented and risk-oriented, respectively.

Table 5: Weights allocation for the inputs and outputs

	Input	Weight		Output	Weight	
					Return-oriented	Risk-oriented
X1	Earning/share	3	Y1	Return	2	1
X2	ROE	2	Y2	Risk	1	2
X3	Growth rate of net income	2				
X4	Asset turnover ratio	1				
X5	Ratio of asset/debt	1				
X6	Liquidity ratio	1				
X7	RSI	2				

The weights in Table 6 represent the relative importance of the indicators, so the constraints can be expressed as follows:

$$\begin{cases} v_1 \geq 1.5v_2;\ v_1 \geq 1.5v_3;\quad v_1 \geq 3v_4; \\ v_1 \geq 3v_5;\ v_1 \geq 3v_6;\quad v_1 \geq 1.5v_7; \\ u_1 \geq 2u_2,\quad or \quad u_1 \leq 0.5u_2 \end{cases} \qquad (5)$$

Using the weight-restricted DEA model with the (5) constraints, the ranking efficiency scores of the twenty-seven stocks are shown in Table 6. The result indicates that, with the return-oriented objective, twelve stocks are first-class efficient; with the risk-oriented objective, only nine stocks are efficient. The number of efficient stocks in both cases is less comparing to the results from fundamental DEA model. This means that the stock selection is stricter by applying the weights. The different weights and objective affect the results also, for example, three of the nonferrous metal stocks satisfy the efficiency criteria in the return-oriented but not all of them are efficient in the risk-oriented, this indicates that the three stocks have less risk tolerance than the other first-class stocks. For different industries, the stocks in nonferrous metal sector have a higher average DEA score than the other sectors. Ignoring the difference in sampling, it conveys the information that the stocks of nonferrous metal have an outstanding performance in 2007, which is proved by the actual market performance.

Table 6: Results from weight-restricted model

sector	stock name	Weight-restricted model (return-oriented)	Weight-restricted model (risk-oriented)	basic model
agriculture	XSGF	0.898	0.892	0.905
	GTZL	0.875	0.775	0.890
	ZLTH	1.000	1.000	1.000
	DFHY	0.618	0.634	0.754
	FLZY	0.824	0.849	0.881
	average	**0.843**	**0.830**	**0.886**
real estates	WKA	0.880	0.851	0.883
	ZSDC	1.000	1.000	1.000
	JRJ	0.855	0.909	0.923
	ZLDC	1.000	0.897	1.000
	QXJS	0.837	0.850	0.866
	BLDC	1.000	1.000	1.000
	average	**0.929**	**0.918**	**0.945**
steel	HDGT	0.816	0.854	0.878
	BGGF	0.872	0.874	1.000
	WGGF	0.793	0.704	0.795
	AGGF	1.000	1.000	1.000
	LGGF	0.761	0.770	0.848
	average	**0.849**	**0.840**	**0.904**
nonferrous metal	ZJHJ	1.000	0.966	1.000
	ZSGF	1.000	0.944	1.000
	TYGY	1.000	1.000	1.000
	ZKYH	1.000	1.000	1.000
	TTGF	1.000	1.000	1.000
	average	**1.000**	**0.982**	**1.000**
electrical equipment	GDDL	0.765	0.670	1.000
	HNGJ	0.828	0.823	1.000
	SZNY	1.000	1.000	1.000
	GDNZ	0.872	0.873	0.885
	PGDQ	0.736	0.779	0.814
	XDGF	1.000	1.000	1.000
	average	**0.867**	**0.857**	**0.950**
Average		**0.897**	**0.886**	**0.938**

4. Conclusions

The results from the return-oriented model are different from the risk-oriented model. In accordance with the different objective, investors can choose different stocks to build up their portfolio. In this study, we only present simple cases with different object-oriented and weight allocation. It could be inferred that the weight-restricted DEA model can meet different risk preference by the weight adjustment, and it provides a promise of application in the portfolio optimization for different investors.

The weight constraint plays an important role in DEA modeling. The comparative result shows that the weight-restricted model is stricter than the fundamental model, so it is more appropriated for the application of strong risk preference. In DEA modeling, the information is static, and the weight allocation process also needs a subjective judgment. Besides the investor's risk preference, the model needs a qualitative analysis on the information of market performance, industry prospect, policies and so on.

Acknowledgments

The study was partially supported by Beijing Jiaotong University under the contract number of 2007RC109 and the Undergraduate Innovative Experimental Program.

Reference

[1] Bruno Solnik (1999), *International Investments,* Addison Wesley Longman, Inc.

[2] Hong Yu Li, Jishan Zhu (2005), "Ranking the Efficiency Performance within a set of Decision Making Units by Data Envelopment Analysis", *International Journal of Information Technology & Decision Making*, 4(3): PP345-357

[3]Antonella Basso, Stefania Funari (2001), "A data envelopment analysis approach to measure the mutual fund performance", *European Journal of Operational Research*, Vol. 135, PP477-492

[4] Guangxi Cao, Jianwei Xia (2007), "DEA Method of Performance Evaluation of Investment Founds", *Mathematics in Practice and Theory*, Vol. 37: PP7-14

[5] Tieniu Dong, Naiding Yang, Jijiao Jiang, Liang Wang (2007), "Performance Evaluation of Open-end Fund Based on Super-efficiency DEA", *Management Review,* Vol. 19, PP11-17

[6] Yihua Chen, Qibing Chen, Yucheng Dong (2007), Guiqing Zhang, "An Empirical Analysis on the Relative Efficiency of the Input-output of Science and Technology Using Improved DEA Model", *Mathematics in Practice and Theory,* Vol. 37, PP38-43

[7] Srinivas Talluri (2000), "Data Envelopment Analysis: Models and Extensions", *Decision Line*, Vol. 5, PP8-11.

[8] Qiang Liu, Zhenquan Zhao (2004), "DEA Method Applied in the Stock Evaluation", *Industrial Technology & Economy,* Vol. 23, PP106-109

[9] Michael C. Knapp (2003), *Financial Accounting: A Focus on Decision Making*, West Publishing Company.